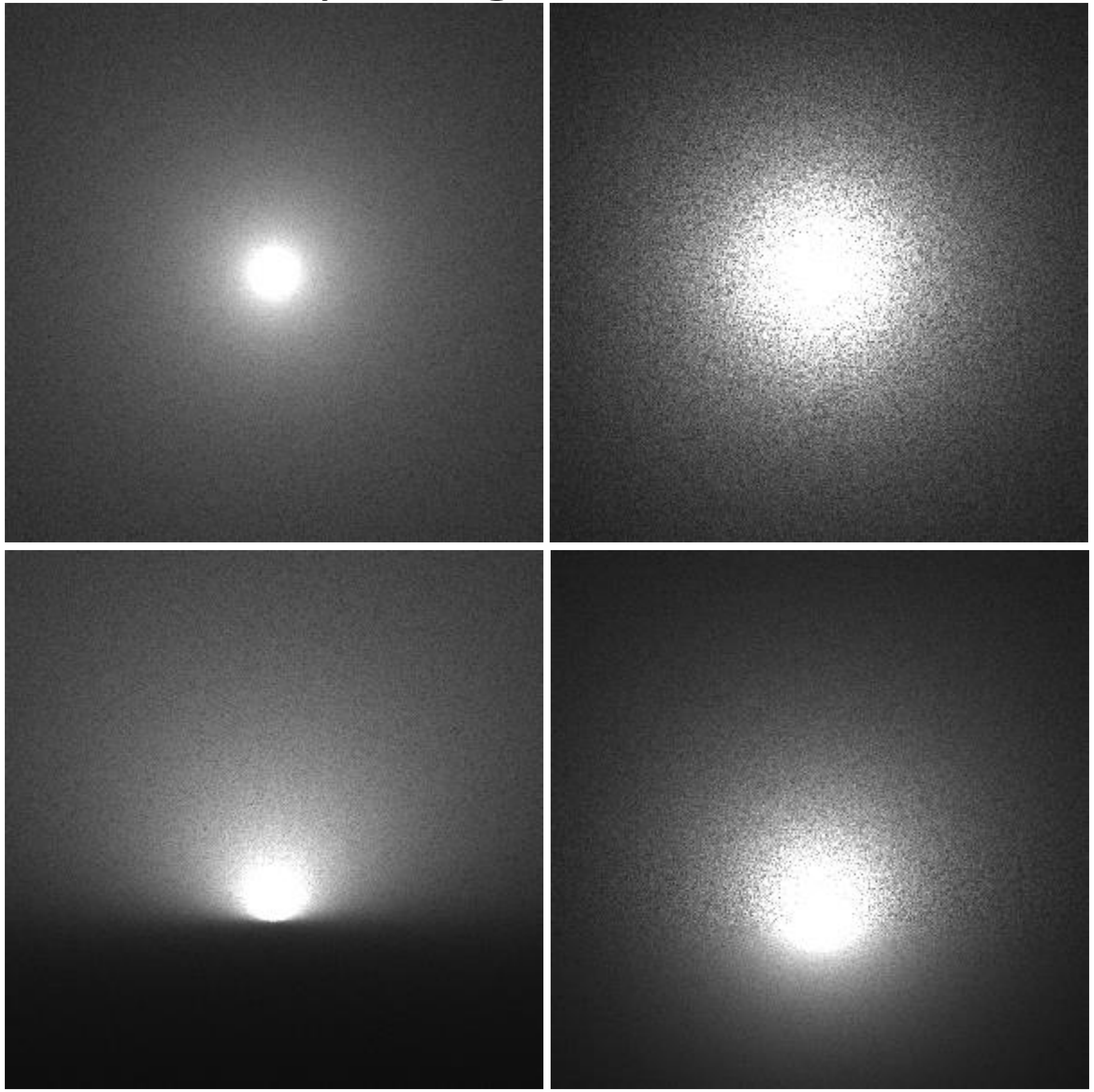
Practical product sampling for single scattering in media

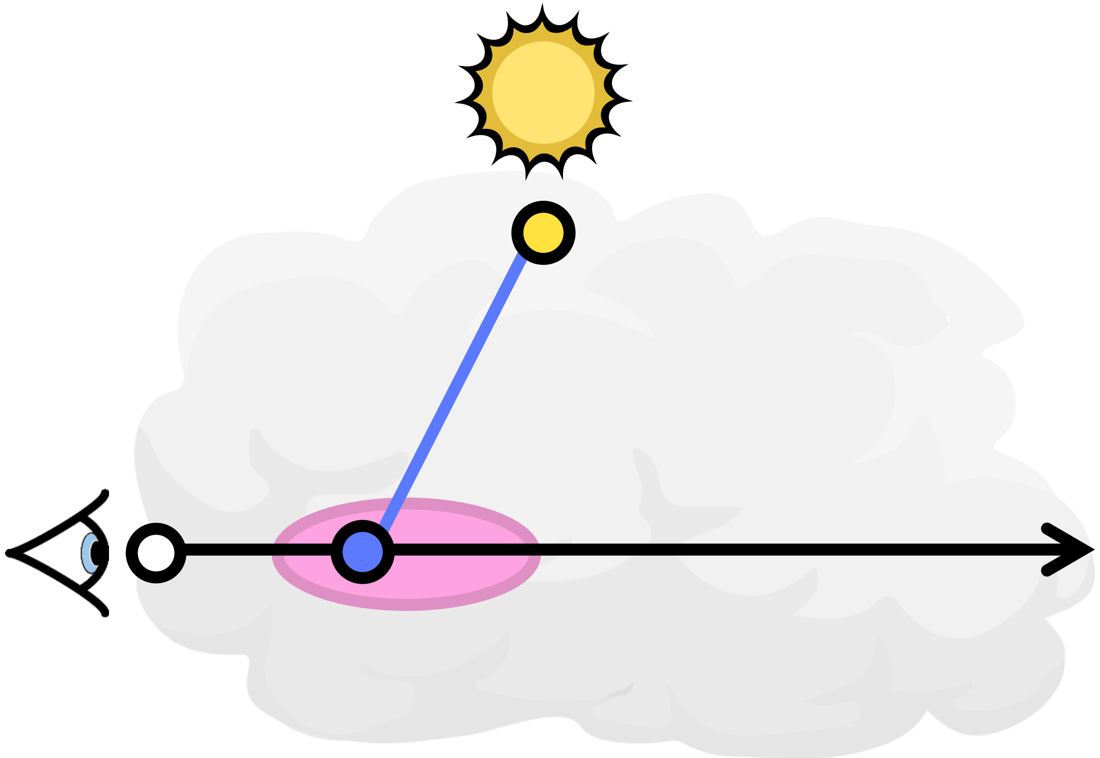
Keven Villeneuve, Adrien Gruson, Iliyan Georgiev, Derek Nowrouzezahrai

Motivation

Motivation

Equi-angular [KF12]

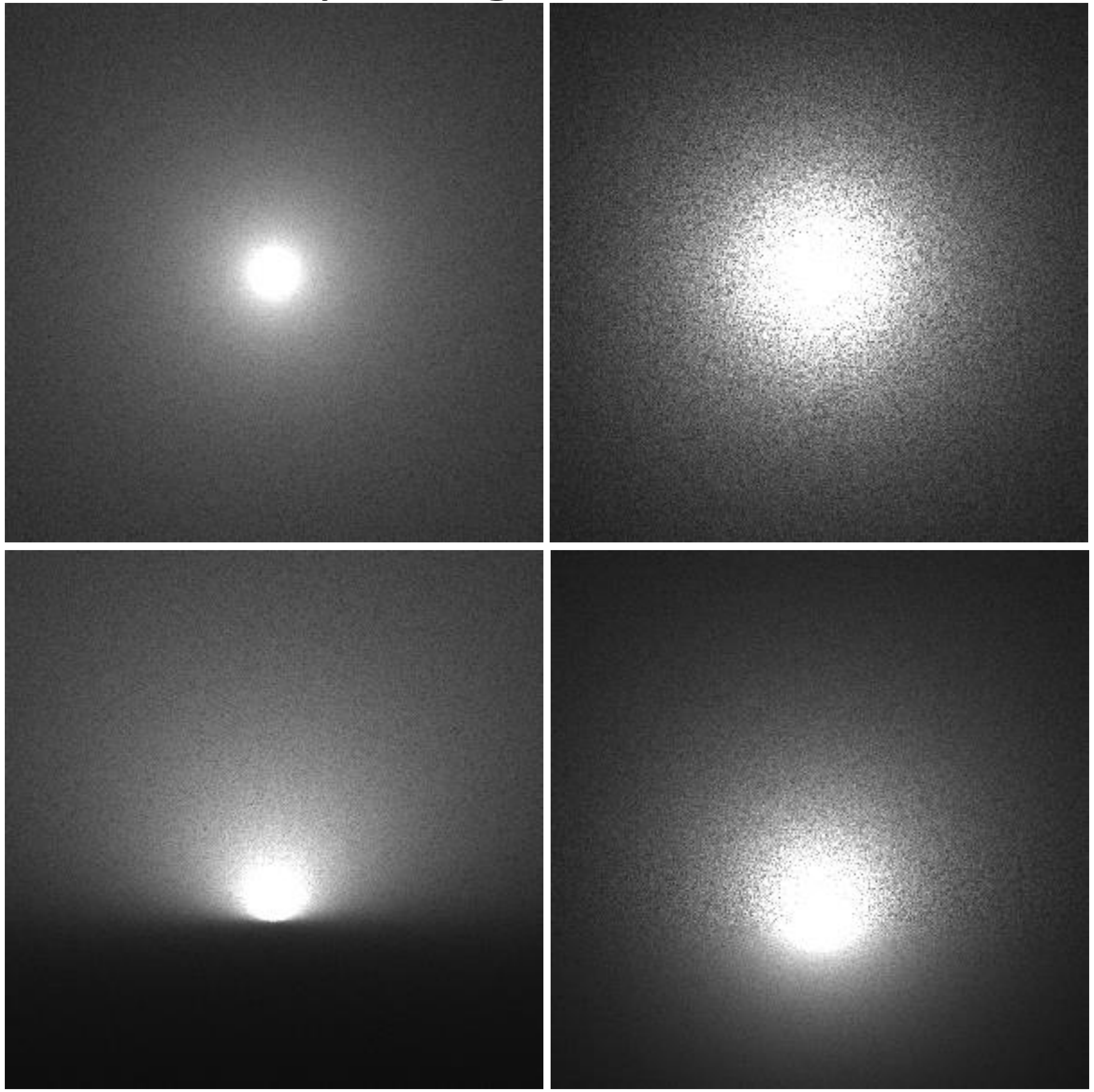




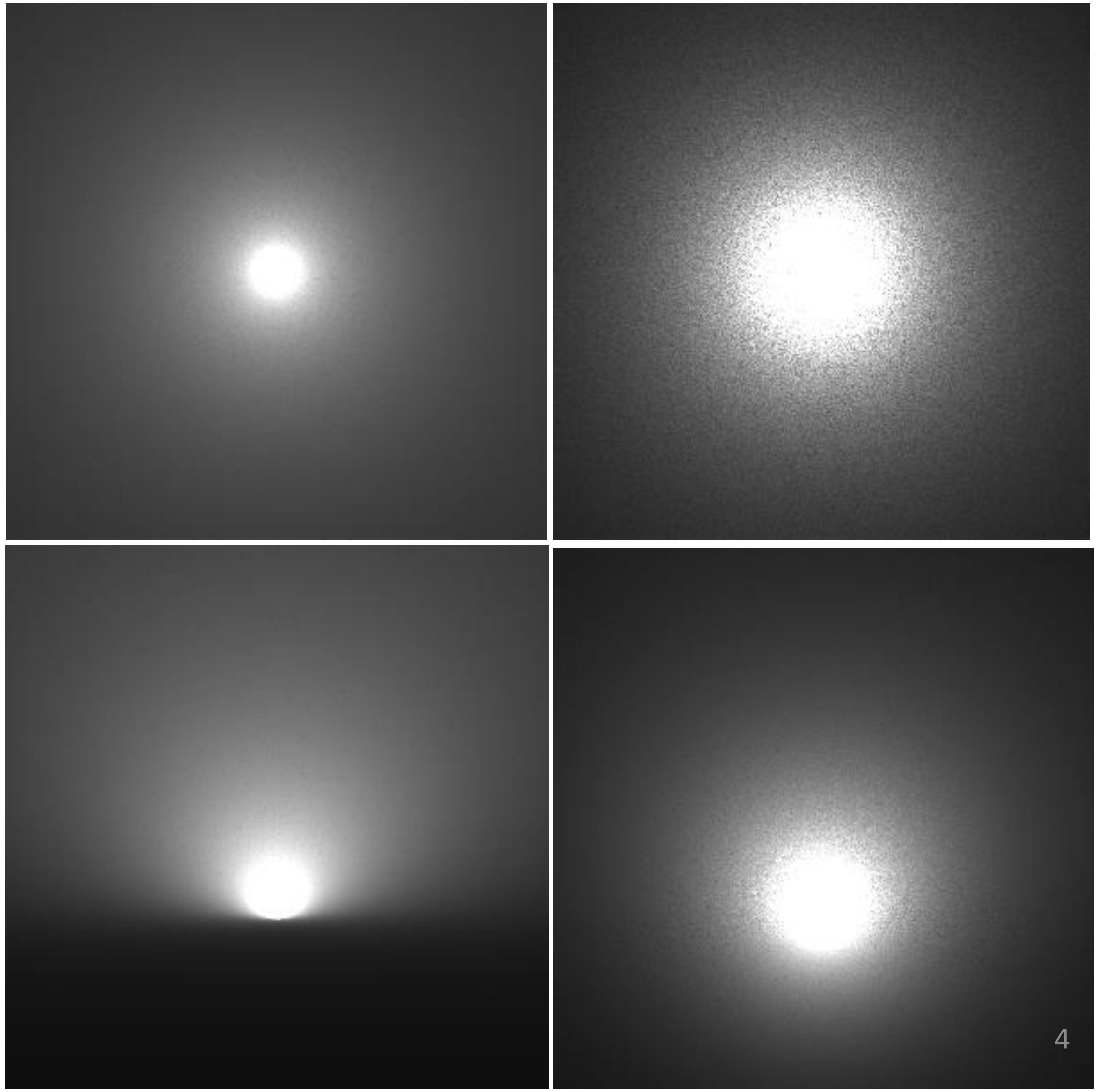
Cosine Foreshortening
Phase function
Transmittance
...

Motivation

Equi-angular [KF12]

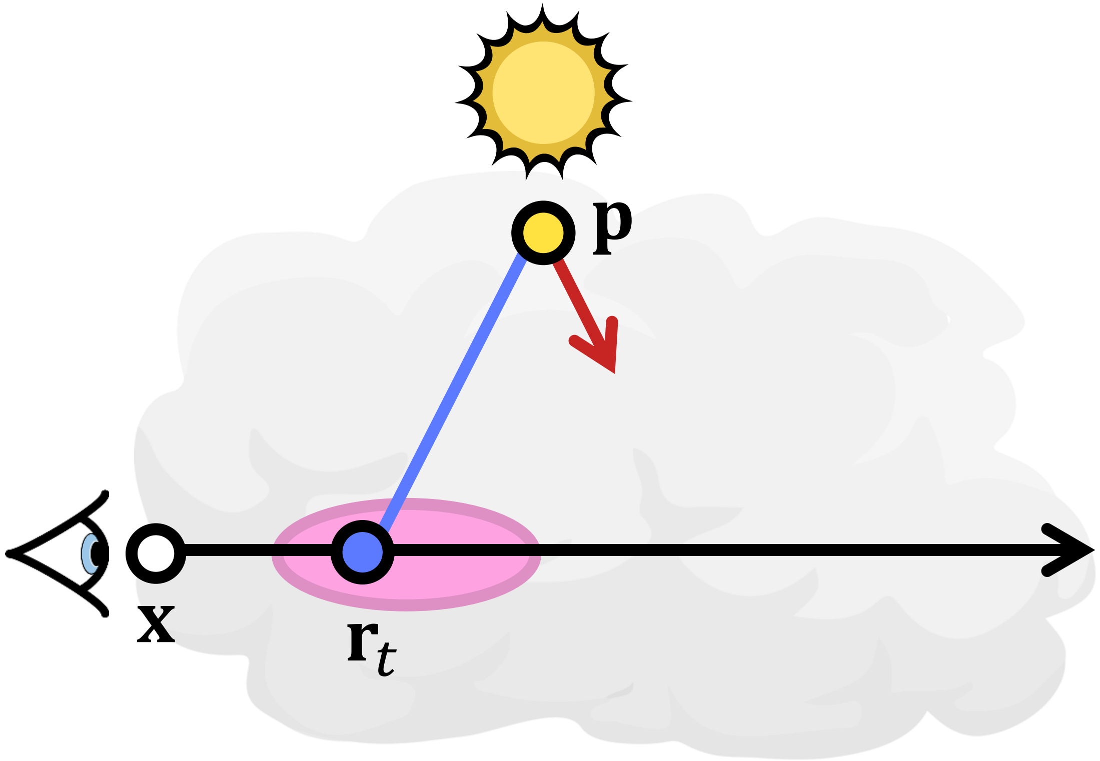


Ours



Background

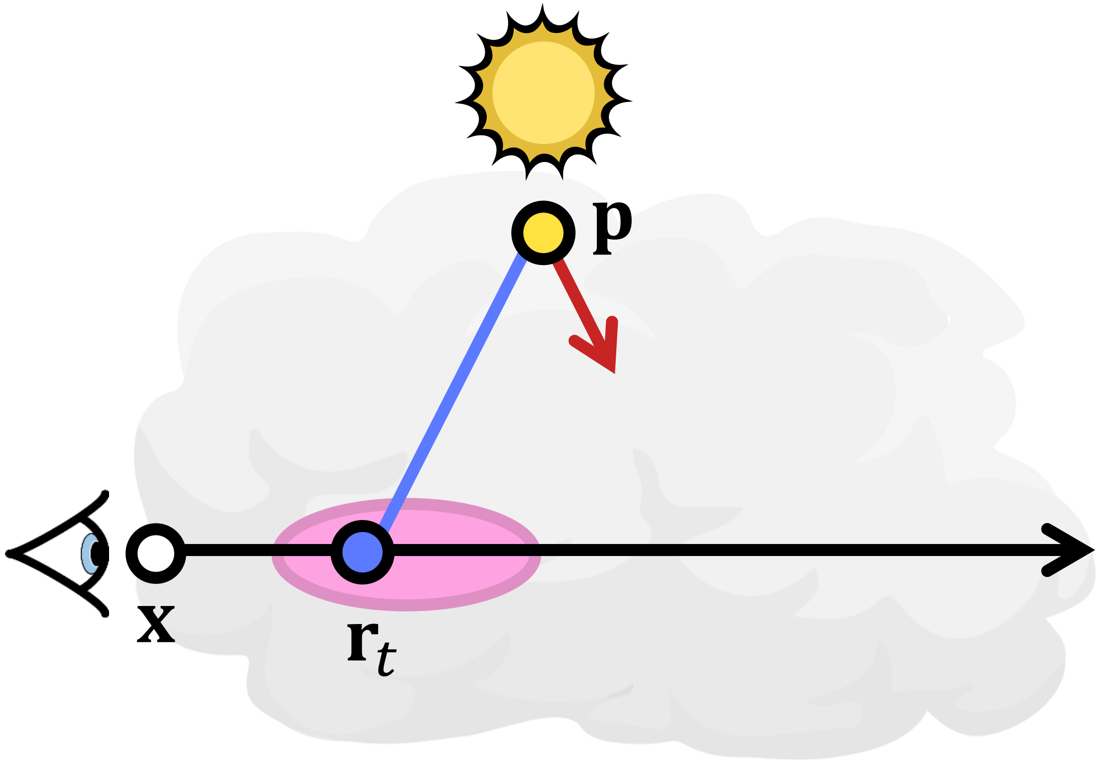
Volumetric Path Tracing



$$L = \int_{t_{\min}}^{t_{\max}} L_e(\mathbf{p}, \mathbf{r}_t)\rho(\mathbf{x}, \mathbf{r}_t, \mathbf{p})T(\mathbf{x}, \mathbf{r}_t)T(\mathbf{r}_t, \mathbf{p})G(\mathbf{r}_t, \mathbf{p})\,\mathrm{d}t$$

Background

Volumetric Path Tracing



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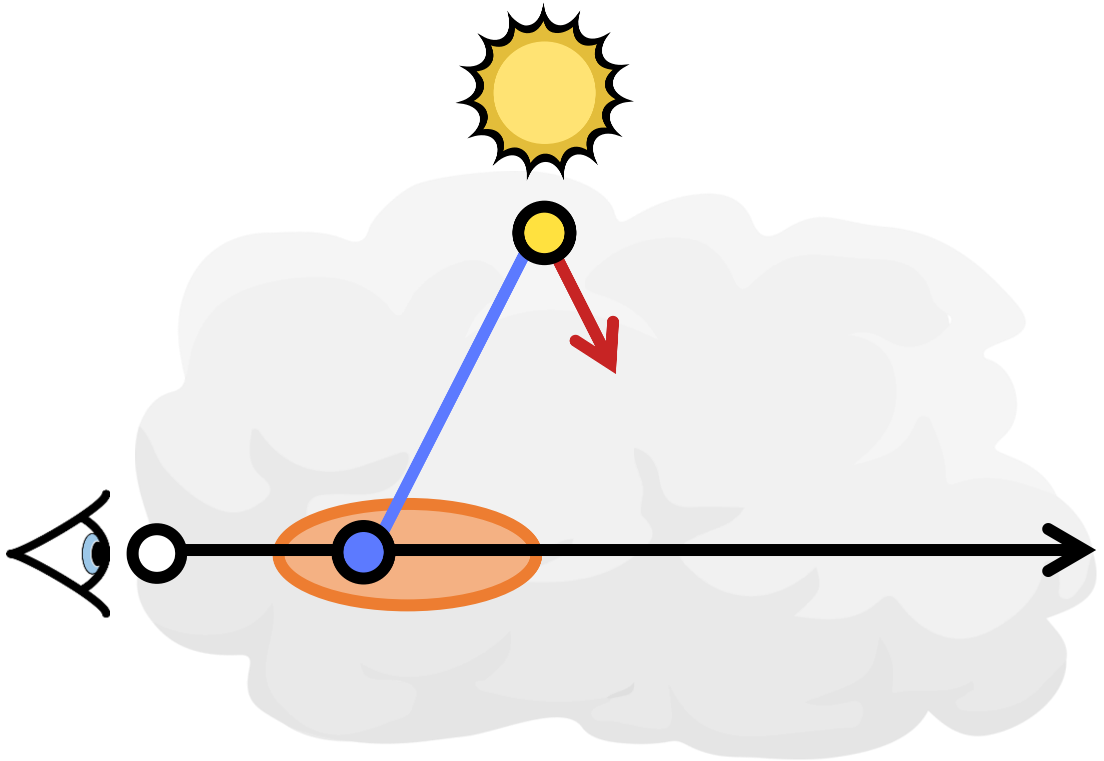
$$F_N = \frac{1}{N}\sum_{i=1}^{N}\frac{f(X_i)}{p(X_i)}$$

Background

Volumetric Path Tracing



$$L = \int_{t_{\min}}^{t_{\max}} \underbrace{L_e(\mathbf{p}, \mathbf{r}_t)\rho(\mathbf{x}, \mathbf{r}_t, \mathbf{p})T(\mathbf{x}, \mathbf{r}_t)T(\mathbf{r}_t, \mathbf{p})G(\mathbf{r}_t, \mathbf{p})}_{f(t)}\,\mathrm{d}t$$

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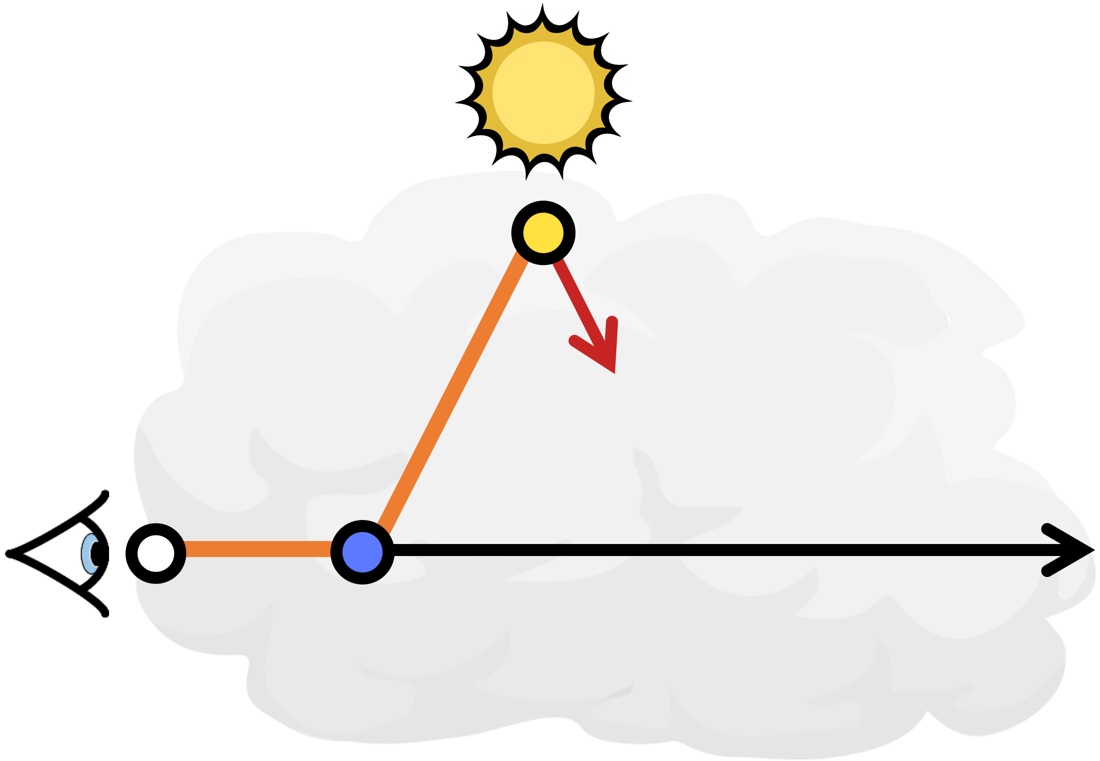

$$L = \int_{t_{\min}}^{t_{\max}} \underbrace{L_e(\mathbf{p}, \mathbf{r}_t)\rho(\mathbf{x}, \mathbf{r}_t, \mathbf{p}) T(\mathbf{x}, \mathbf{r}_t) T(\mathbf{r}_t, \mathbf{p}) G(\mathbf{r}_t, \mathbf{p})}_{f(t)} \, \mathrm{d}t$$

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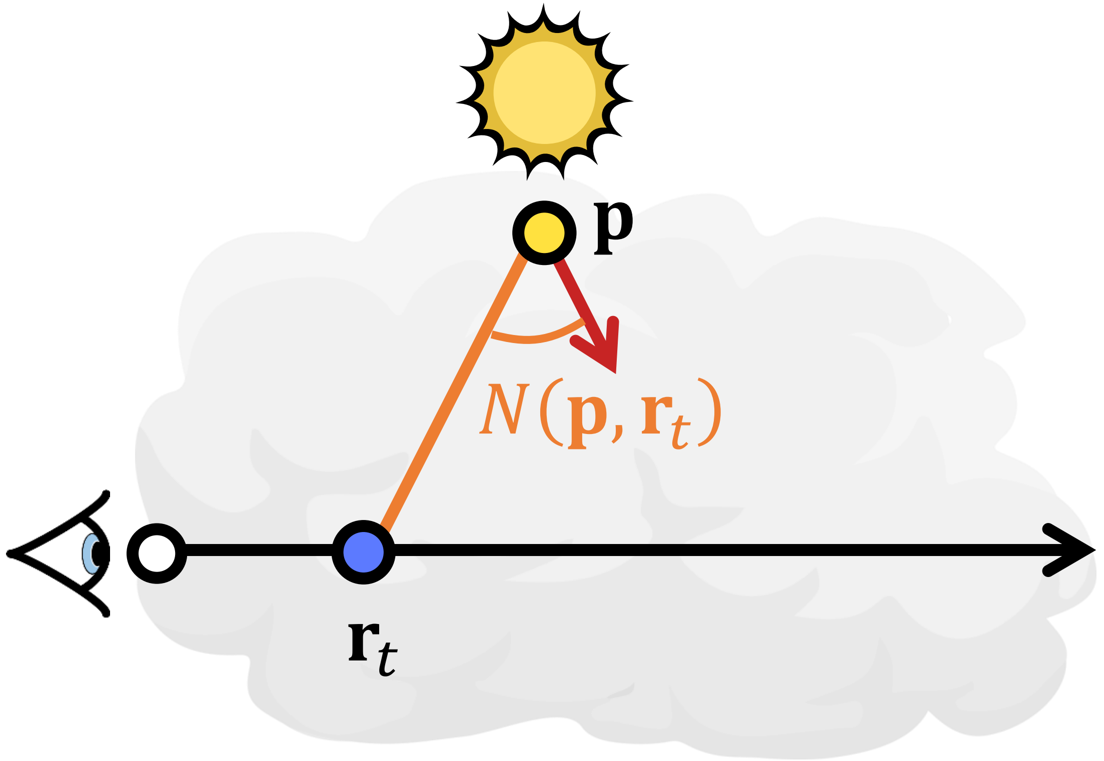
Volumetric Path Tracing



$$L = \int_{t_{\min}}^{t_{\max}} \underbrace{L_e(\mathbf{p}, \mathbf{r}_t)\rho(\mathbf{x}, \mathbf{r}_t, \mathbf{p})T(\mathbf{x}, \mathbf{r}_t)T(\mathbf{r}_t, \mathbf{p})G(\mathbf{r}_t, \mathbf{p})}_{f(t)}\,\mathrm{d}t$$

$$F_N = \frac{1}{N}\sum_{i=1}^{N}\frac{f(X_i)}{p(X_i)}$$

$$G(\mathbf{r}_t, \mathbf{p}) = \frac{N(\mathbf{p}, \mathbf{r}_t)}{\|\mathbf{p} - \mathbf{r}_t\|^2}$$

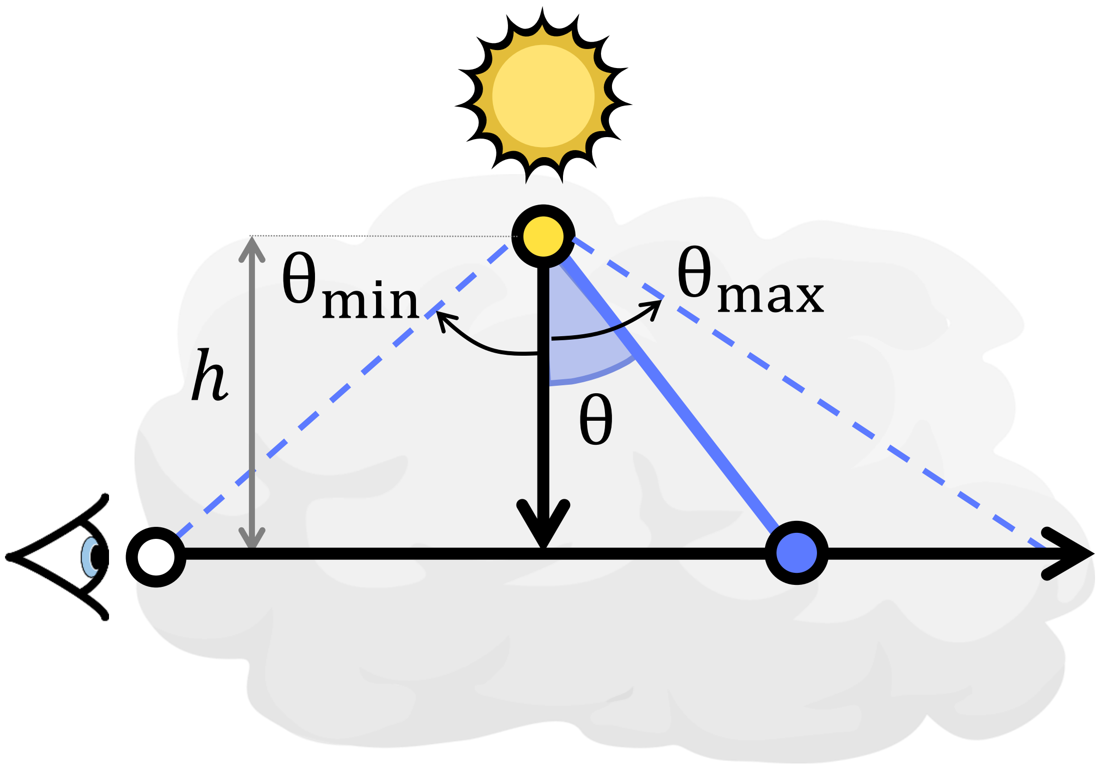
Related Work

Equi-angular Sampling [Kulla et Fajardo 2012]

$$L = \frac{L_e}{h} \int_{\theta_{min}}^{\theta_{max}} \rho(\theta) T(\theta) N(\theta) d\theta$$

$$\langle L \rangle = \frac{L_e}{h} \cdot \frac{\rho(\theta) T(\theta) N(\theta)}{p(\theta)}$$

$$p(\theta) = \frac{1}{\theta_{max} - \theta_{min}}$$



Related Work

Equi-angular Sampling [Kulla et Fajardo 2012]

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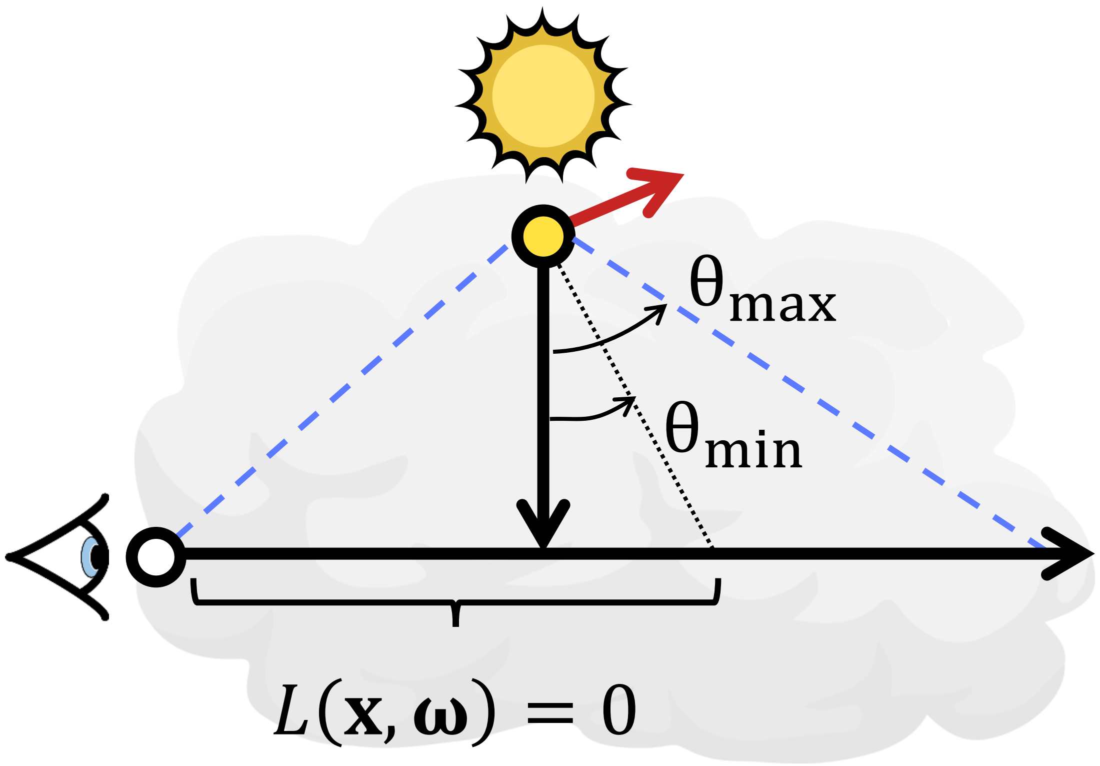
$$p(\theta) = \frac{1}{\theta_{max} - \theta_{min}}$$

Clamped cosine: $\theta_{max}, \theta_{min}$

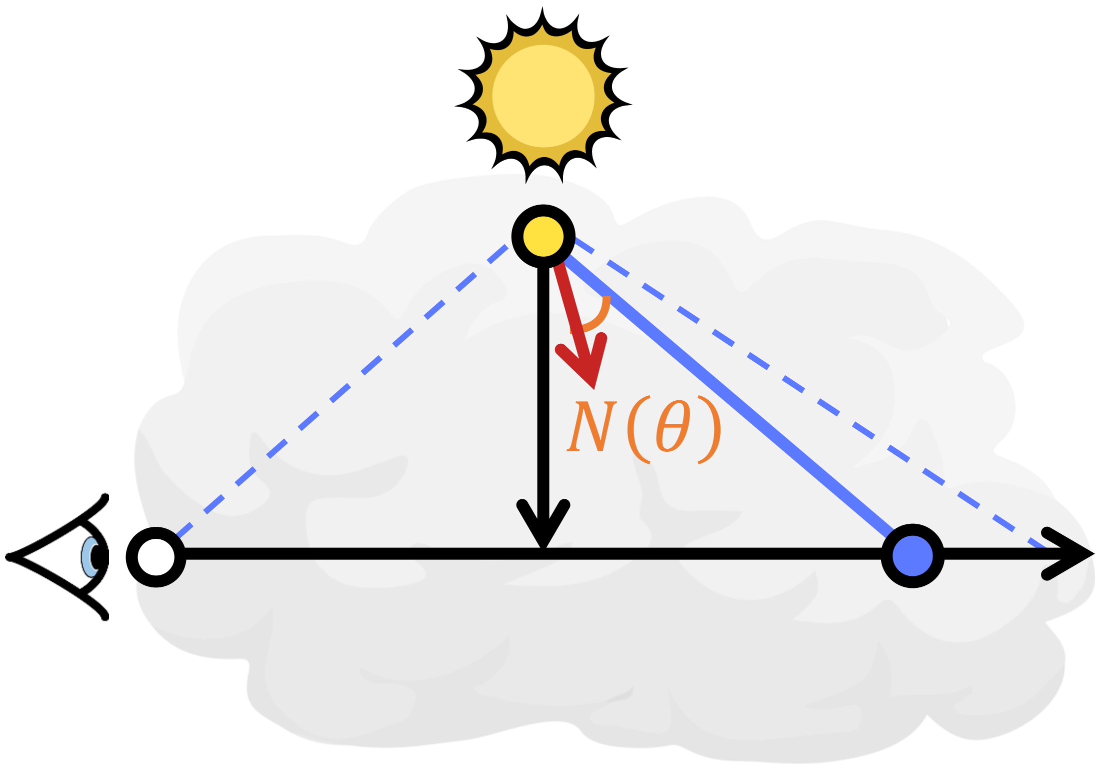


Analytical Point-normal Sampling

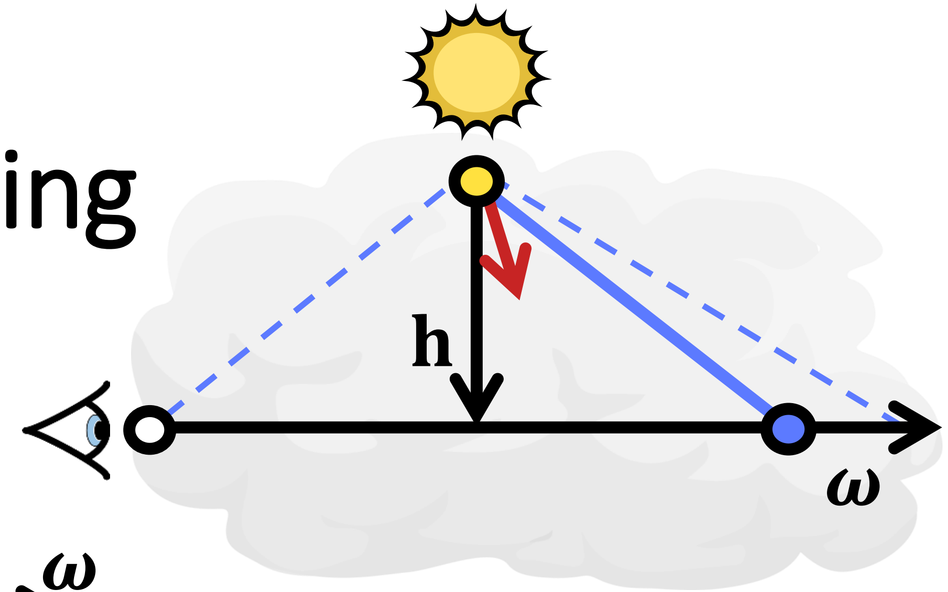
$$L = \frac{L_e}{h} \int_{\theta_{min}}^{\theta_{max}} \rho(\theta) T(\theta) N(\theta) d\theta$$

$$\langle L \rangle = \frac{L_e}{h} \cdot \frac{\rho(\theta) T(\theta) N(\theta)}{p(\theta)}$$

$$p(\theta) \propto N(\theta)$$



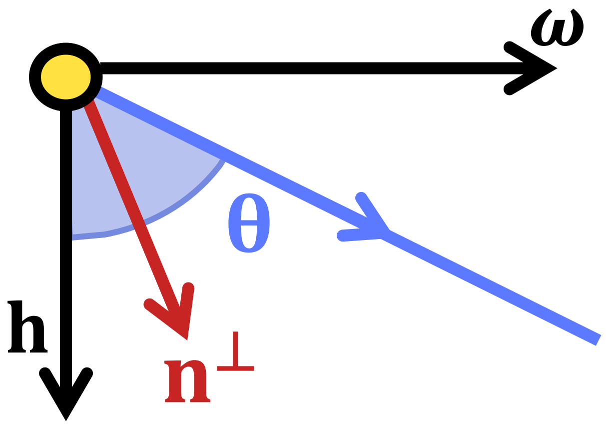
Analytical Point-normal Sampling



Parametrizing $N(\theta)$ in local frame

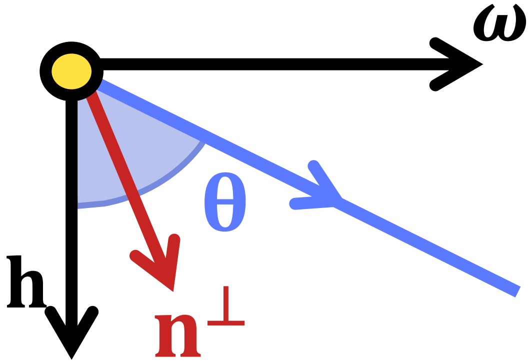
$$\boldsymbol{\theta} = (\cos\theta, \sin\theta)$$

$$\mathbf{n}^{\perp} = (\mathbf{n}\cdot\mathbf{h}, \mathbf{n}\cdot\boldsymbol{\omega})$$



$$N(\theta) = \boldsymbol{\theta}\cdot\mathbf{n}^{\perp} = (\mathbf{n}\cdot\mathbf{h})\cos\theta + (\mathbf{n}\cdot\boldsymbol{\omega})\sin\theta$$

Analytical Point-normal Sampling



$$p_N(\theta) = a\cos\theta + b\sin\theta$$

PDF

Integrate

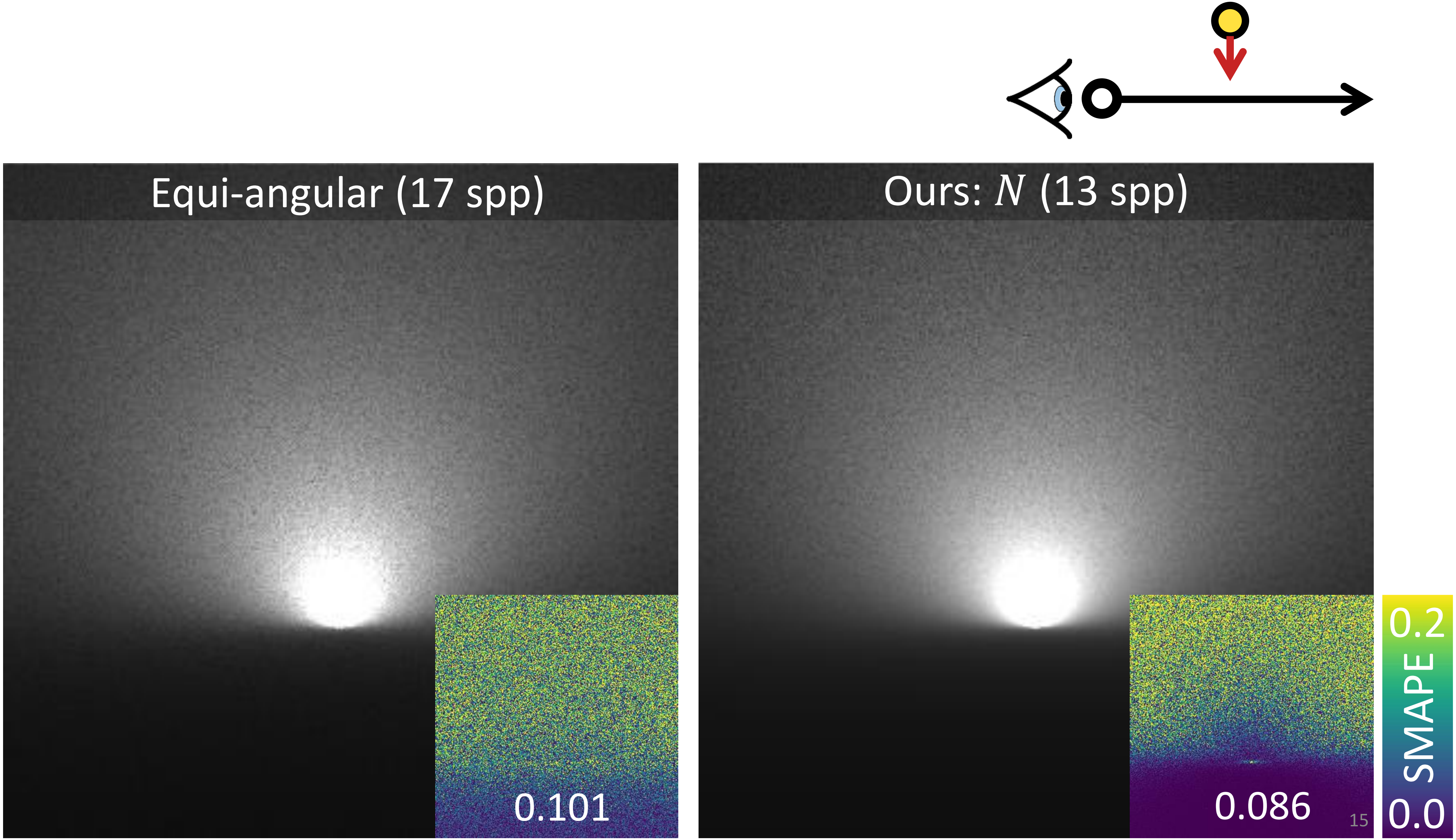
$$P_N(\theta) = a\,(\sin\theta - \sin\theta_{\min}) - b(\cos\theta - \cos\theta_{\min})$$

CDF

Invert

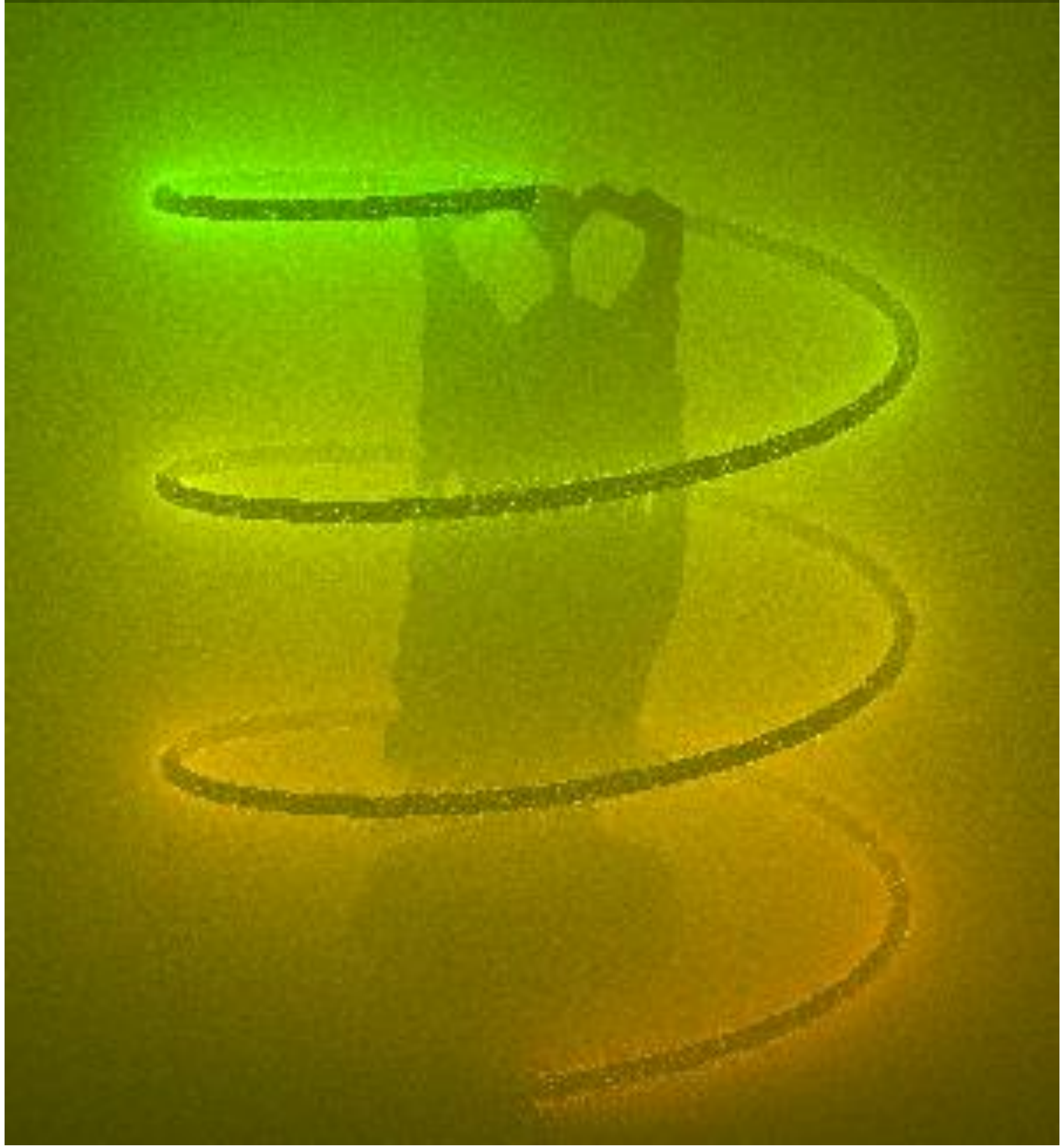
$$\theta = \arctan\left(\frac{|a|c(\xi) \pm \operatorname{sgn}(a)\,bd}{-bc(\xi) \pm d|a|}\right) \qquad \xi\sim[0,1)$$

Analytical sampling

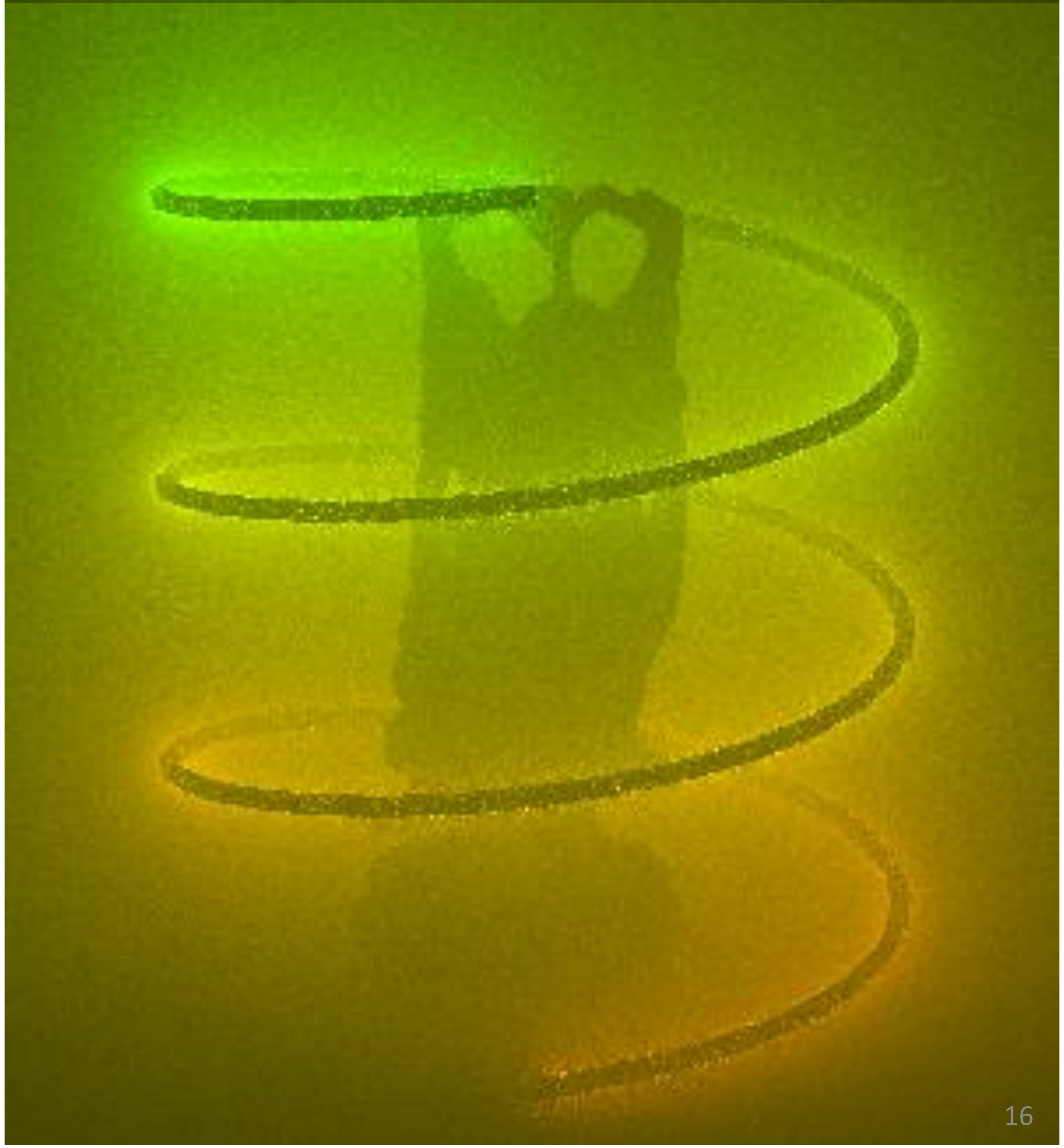


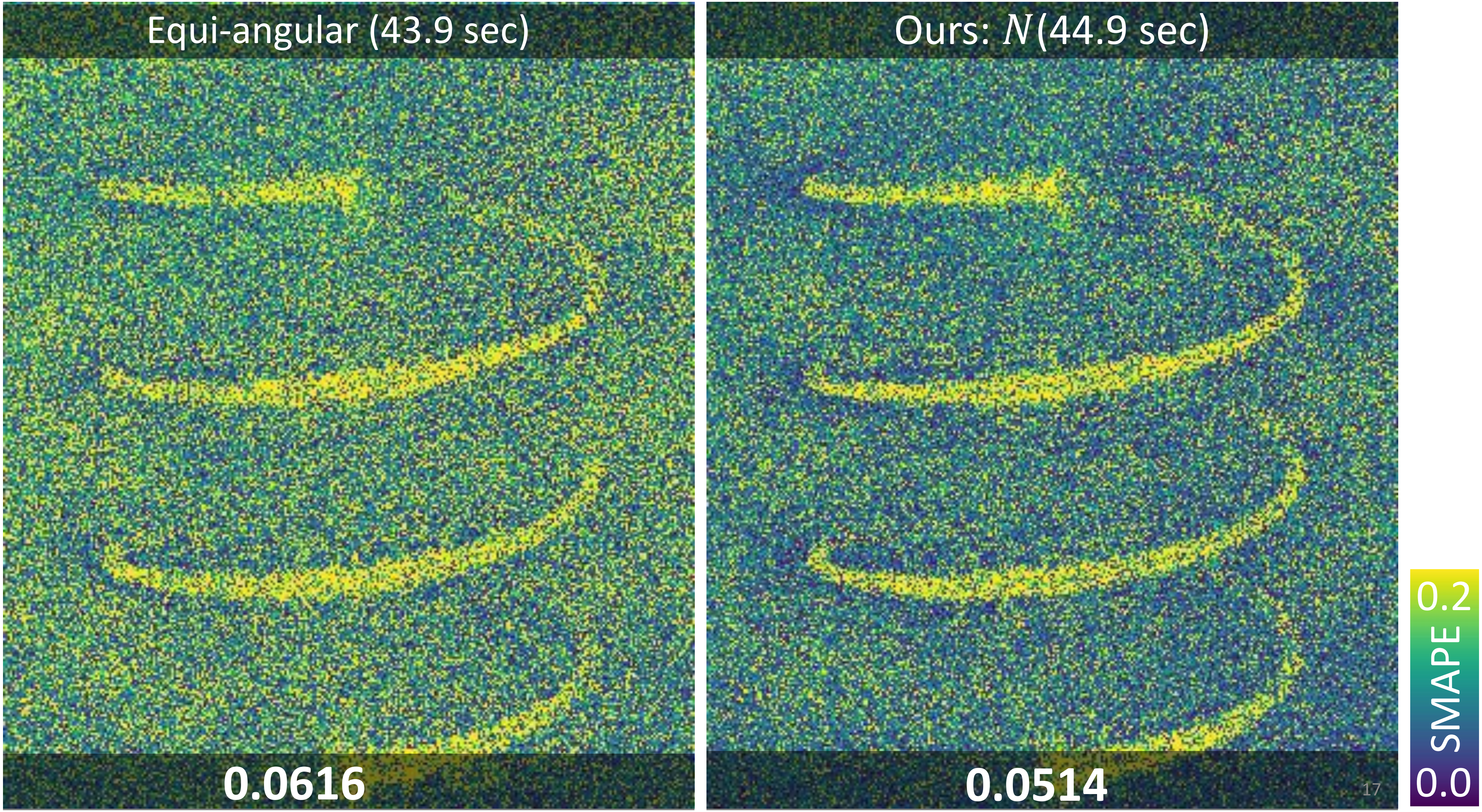
Equi-angular (17 spp)
0.101
Ours: N (13 spp)
0.086
0.2
SMAPE
0.0

Equi-angular (43.9 sec)



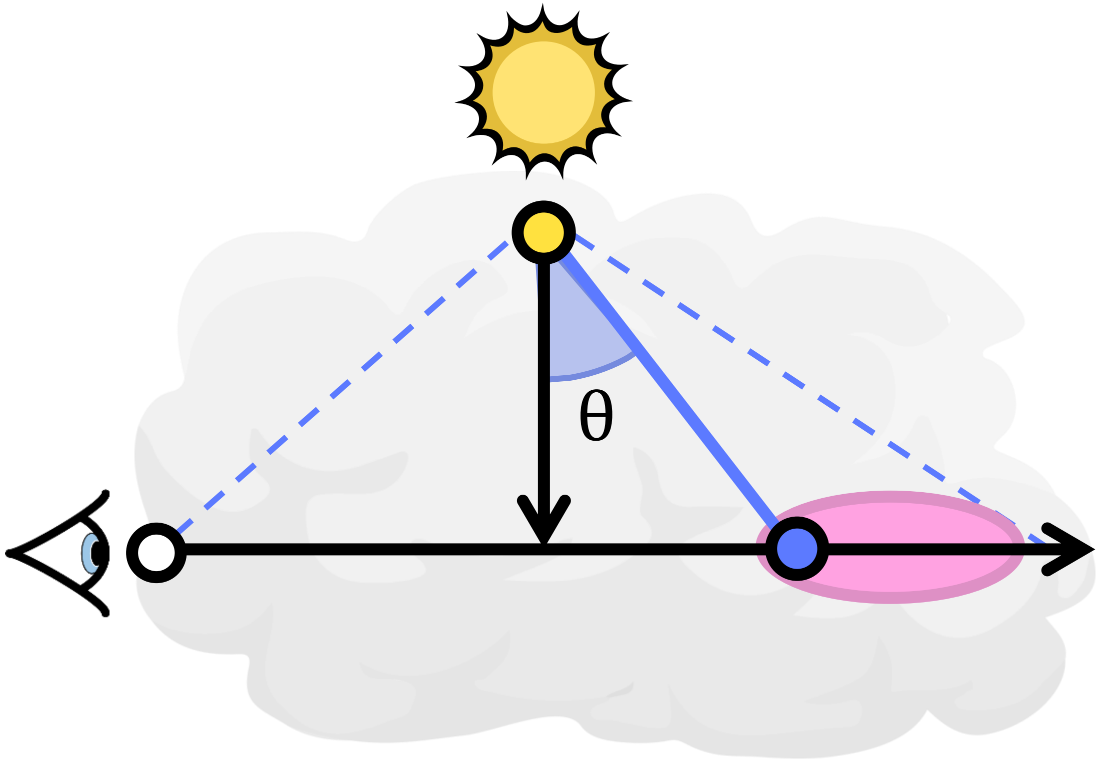
Ours: N(44.9 sec)




Equi-angular (43.9 sec)
Ours: N(44.9 sec)
0.0616
0.0514
0.2
SMAPE
0.0

Remaining terms

$$L = \frac{L_e}{h} \int_{\theta_{min}}^{\theta_{max}} \rho(\theta) T(\theta) N(\theta) d\theta$$



Remaining terms

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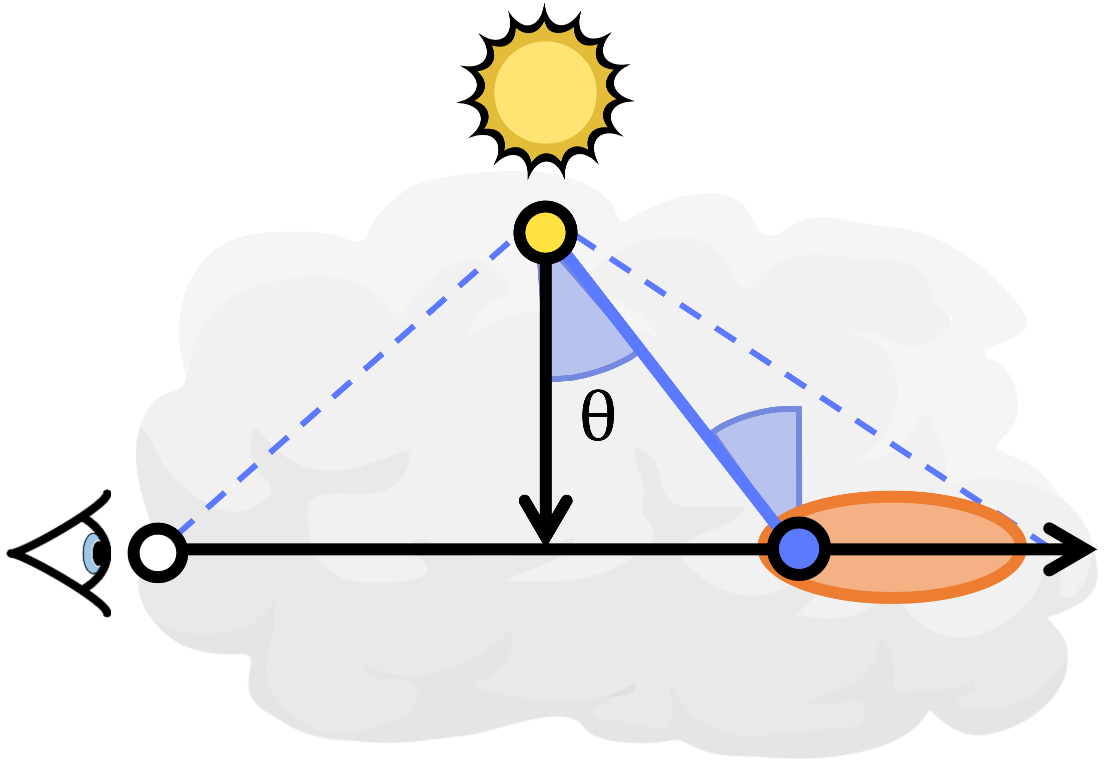
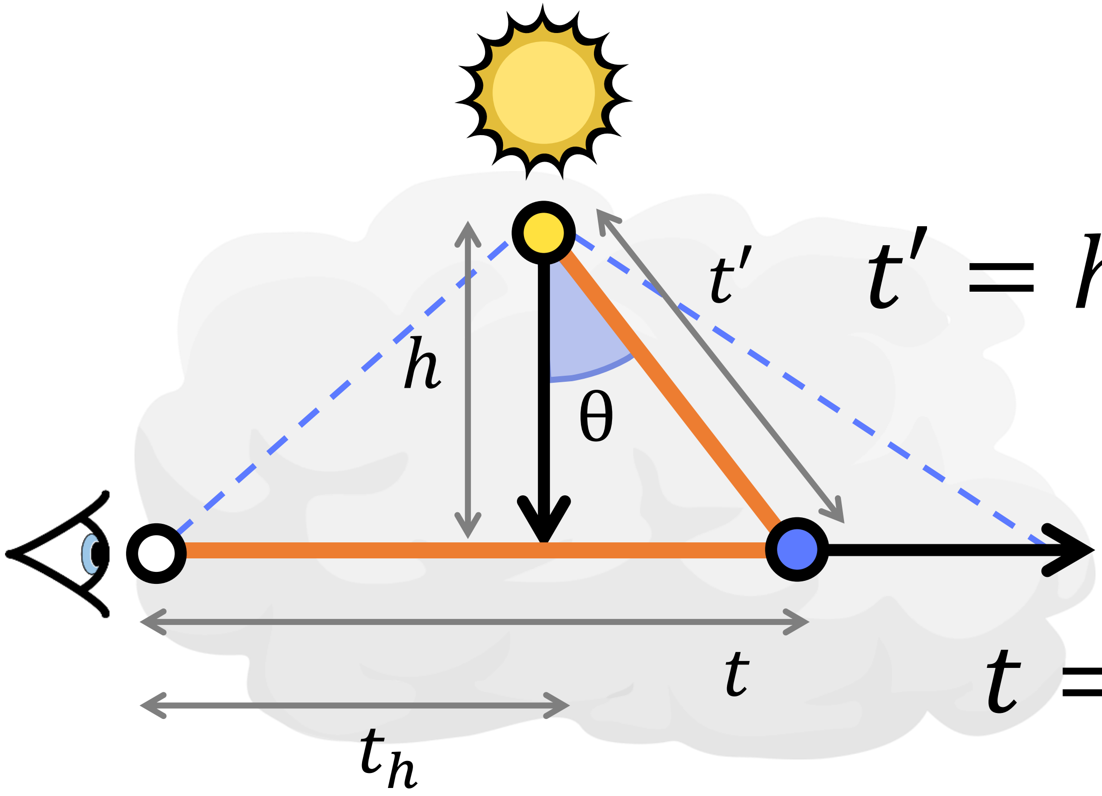


$$\theta_{hg} = \theta + \frac{\pi}{2}$$

Remaining terms

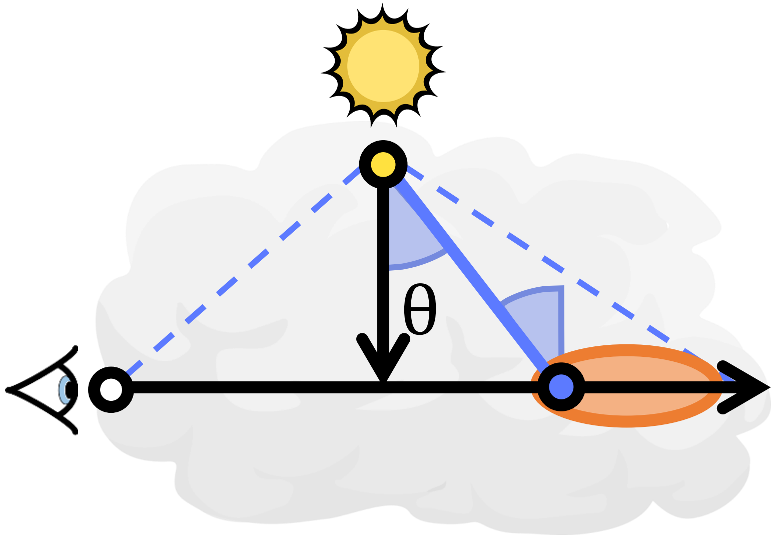
$$L = \frac{L_e}{h} \int_{\theta_{min}}^{\theta_{max}} \rho(\theta) T(\theta) N(\theta) d\theta$$



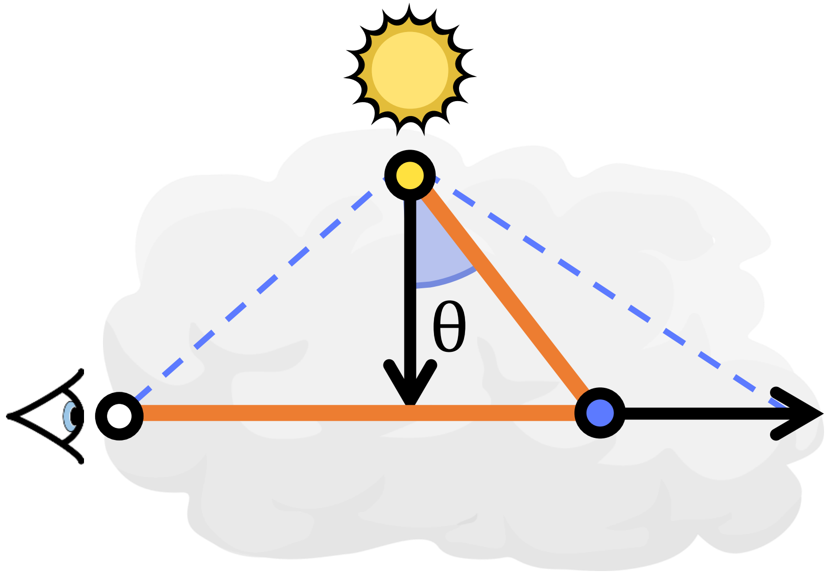
$$t' = h \sec(\theta)$$

$$t = h \tan(\theta) + t_h$$

Remaining terms



$\rho(\theta)$



$T(\theta)$

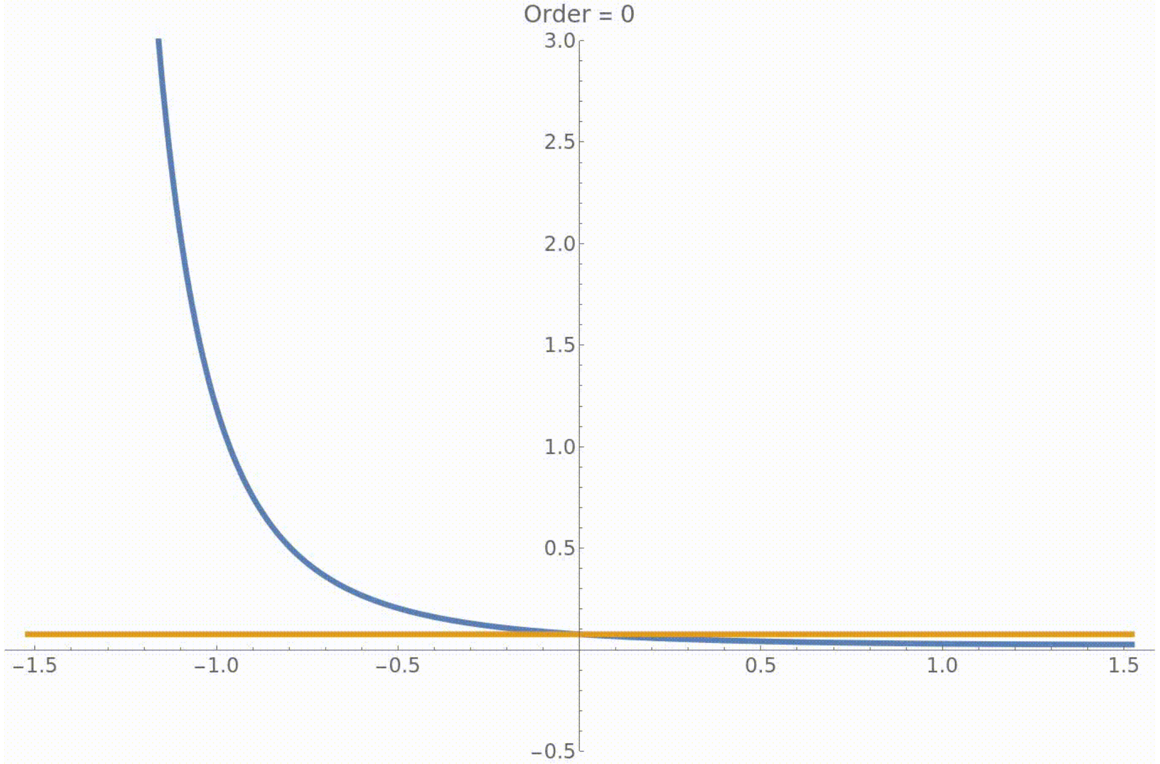
Integrate

Polynomial Approximation $\xrightarrow{\text{Integrate}}$ Polynomial $\xrightarrow{\text{Invert (Numerical)}}$ Sampling

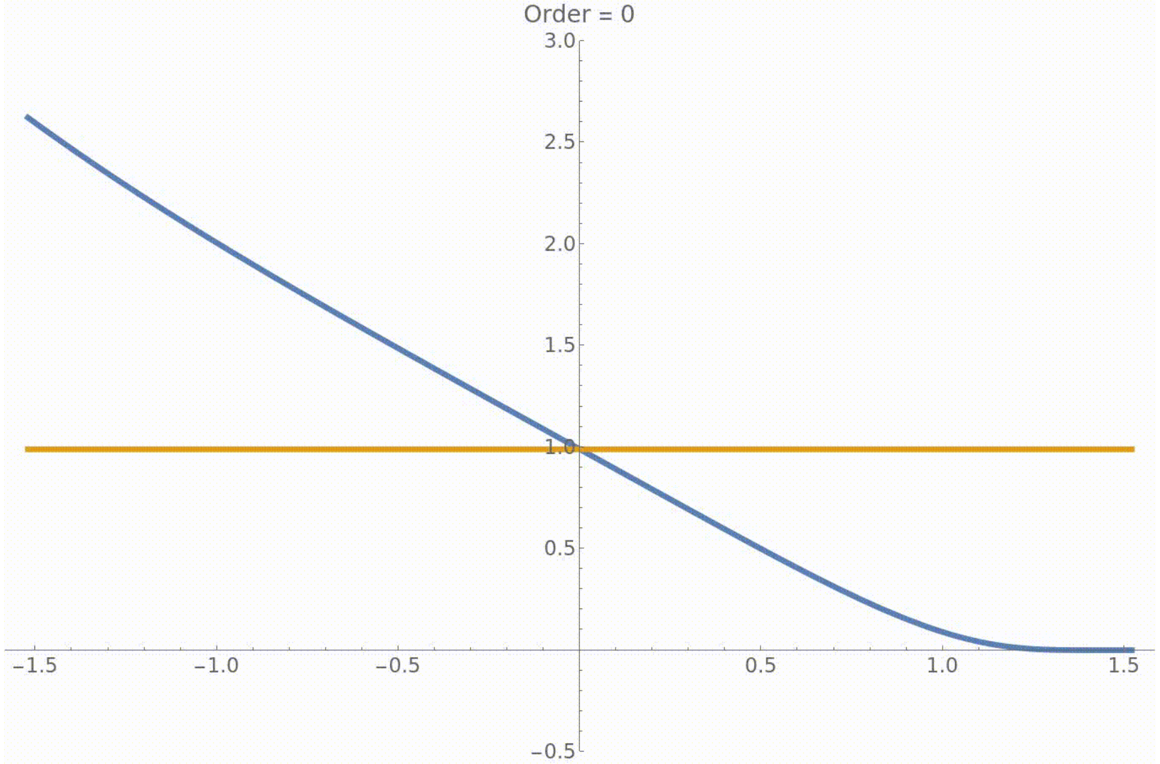
Approximated Transmittance/Phase function

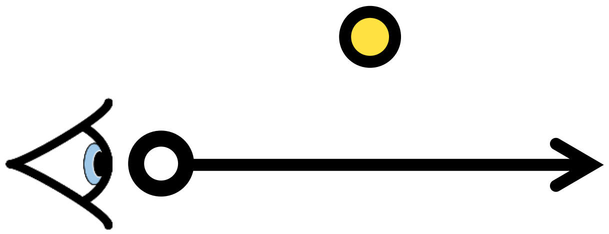
Taylor Expansion

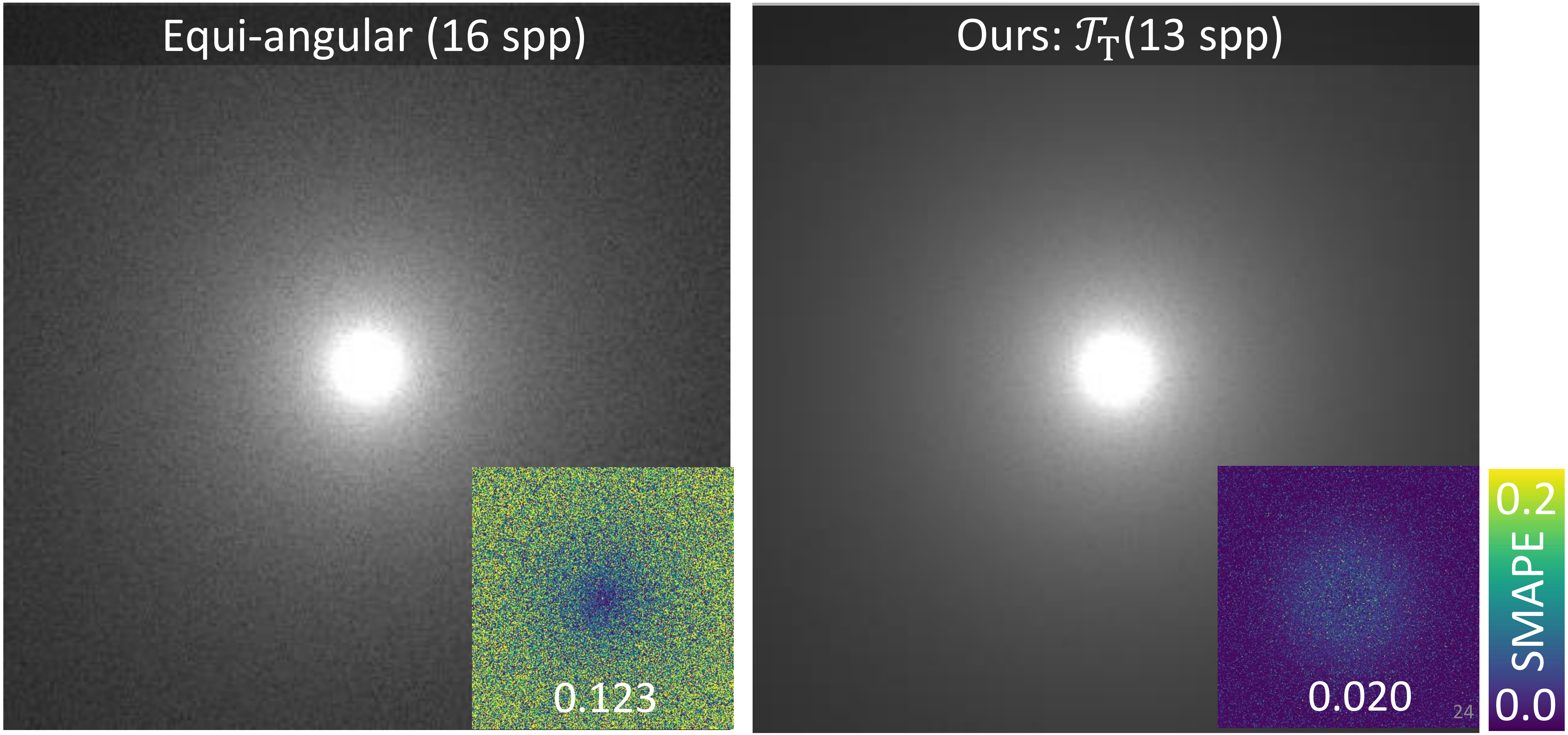
$\rho(\theta)$



$T(\theta)$







Equi-angular (16 spp)
0.123
Ours: $\mathcal{T}_{\mathrm{T}}$(13 spp)
0.020
0.2
SMAPE
0.0

Approximated Product Sampling

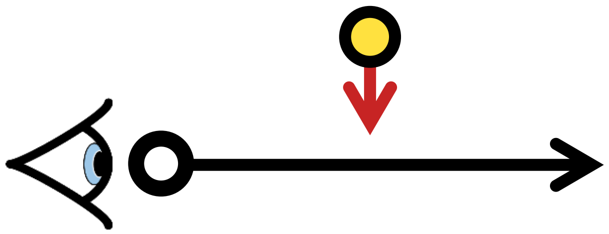
$p(\theta) \propto \mathcal{T}_\rho(\theta)N(\theta)$ or $p(\theta) \propto \mathcal{T}_T(\theta)N(\theta)$

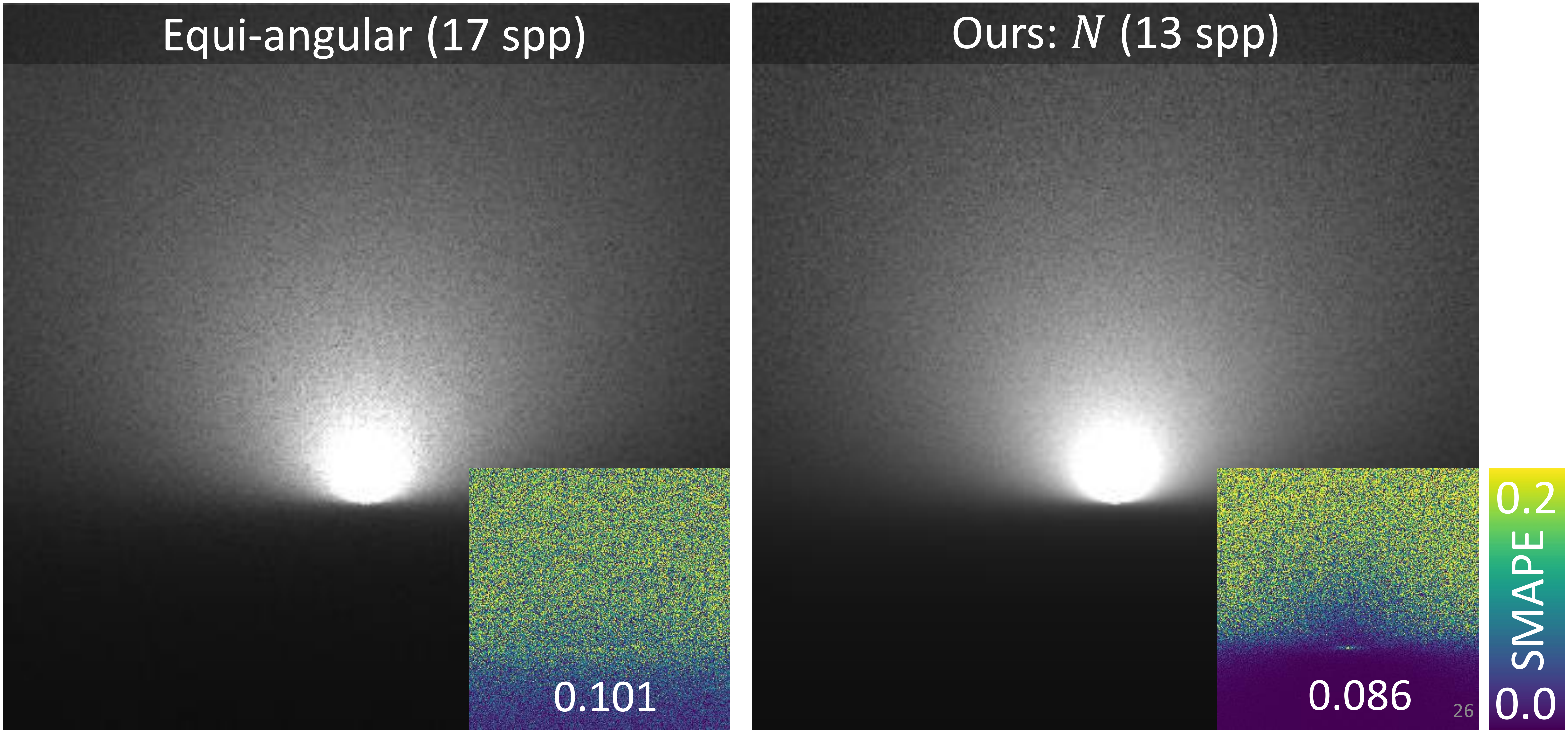
$\Downarrow$ Integrate

$P(\theta)$

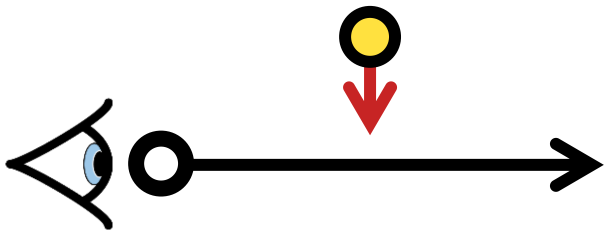
$\Downarrow$ Invert (Numerical)

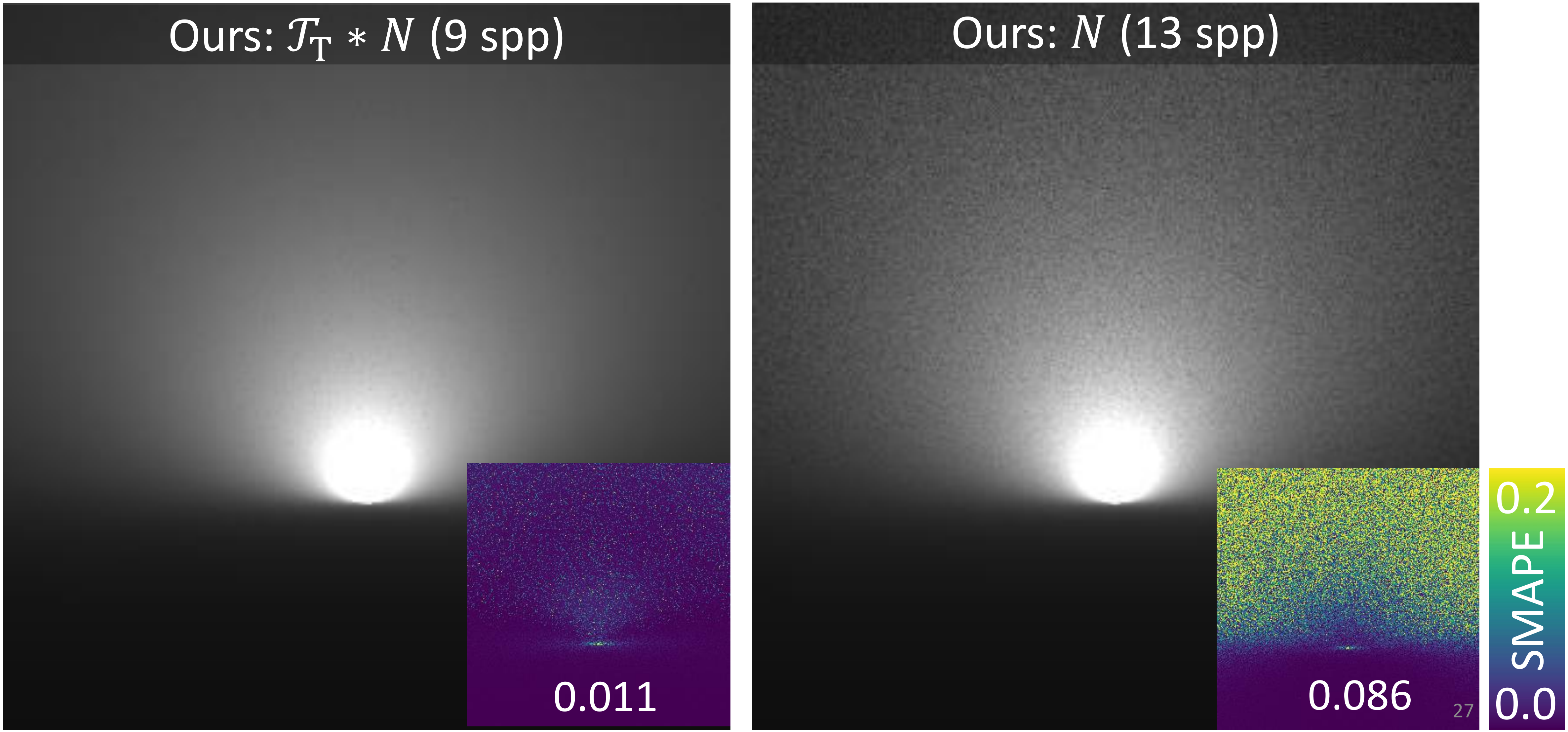
Semi-analytical
Sampling



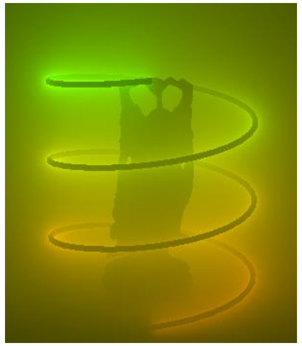


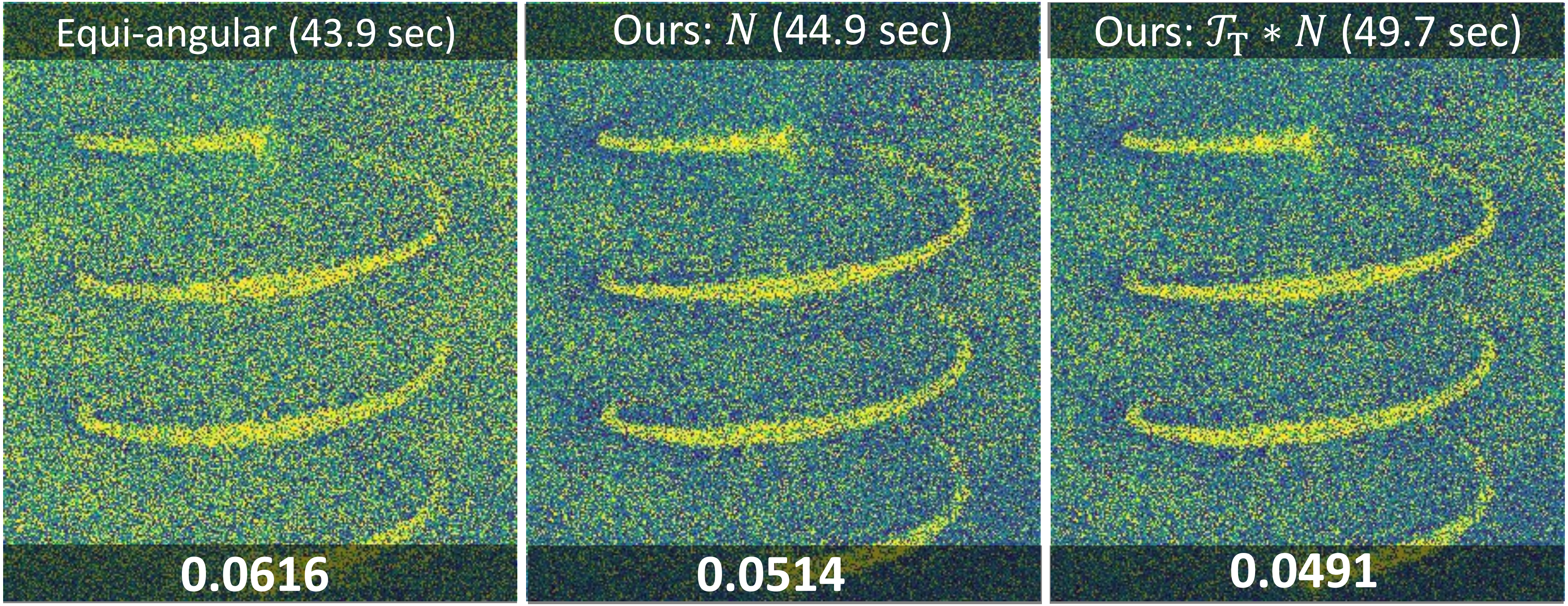
Equi-angular (17 spp)
0.101
Ours: N (13 spp)
0.086
0.2
SMAPE
0.0





Ours: $\mathcal{J}_{\mathrm{T}} * N$ (9 spp)
0.011
Ours: N (13 spp)
0.086
0.2
SMAPE
0.0



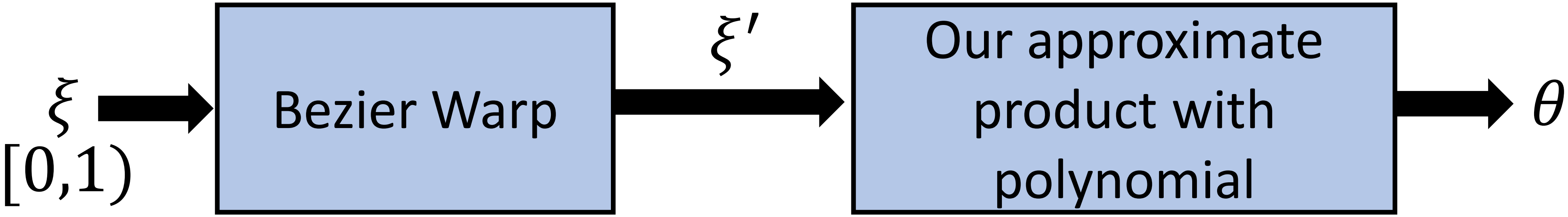

Equi-angular (43.9 sec)
Ours: N (44.9 sec)
Ours: $\mathcal{T}_\mathrm{T} * N$ (49.7 sec)
0.0616
0.0514
0.0491


0.0
SMAPE
0.2

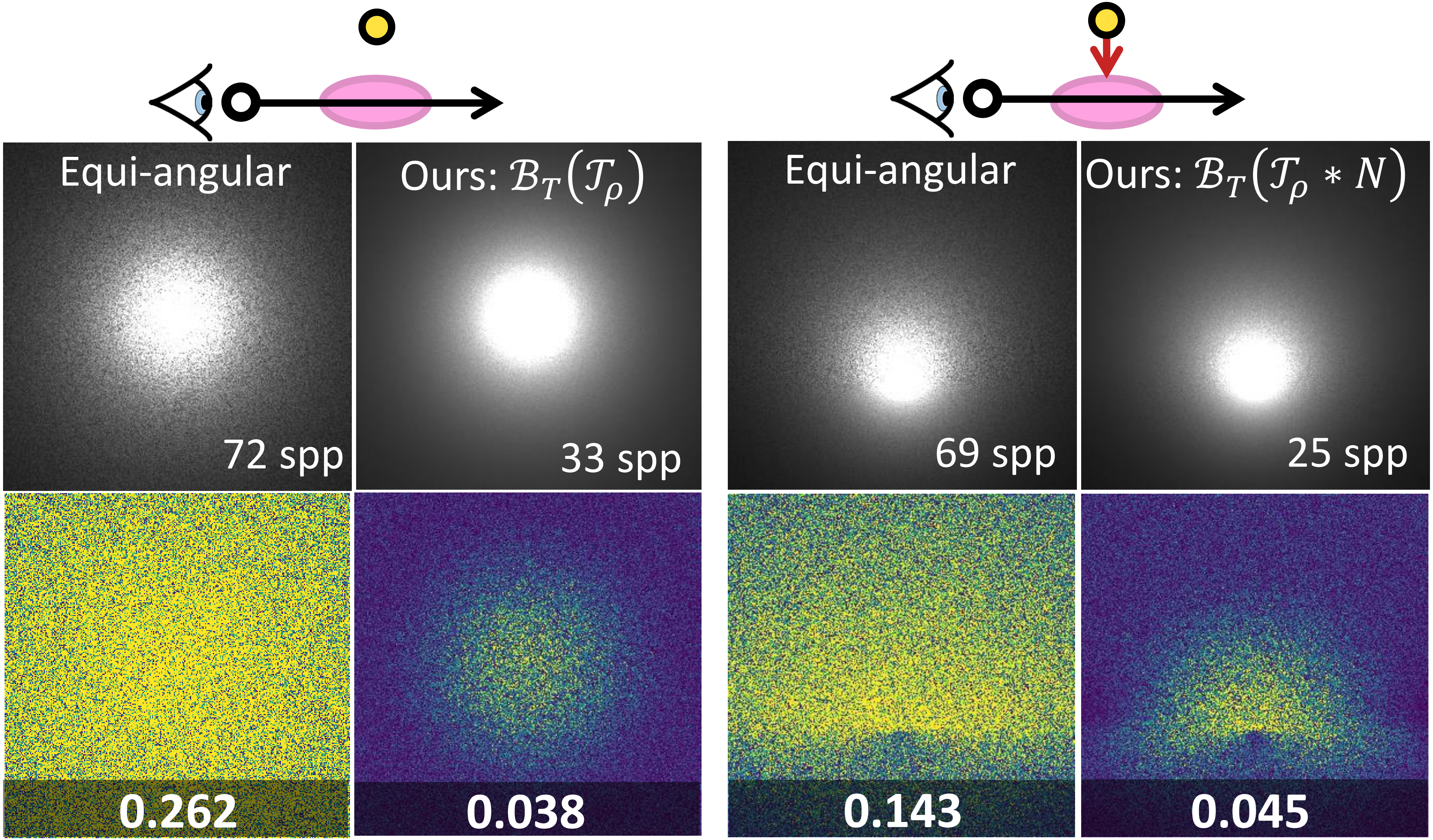
Approximated Product Sampling

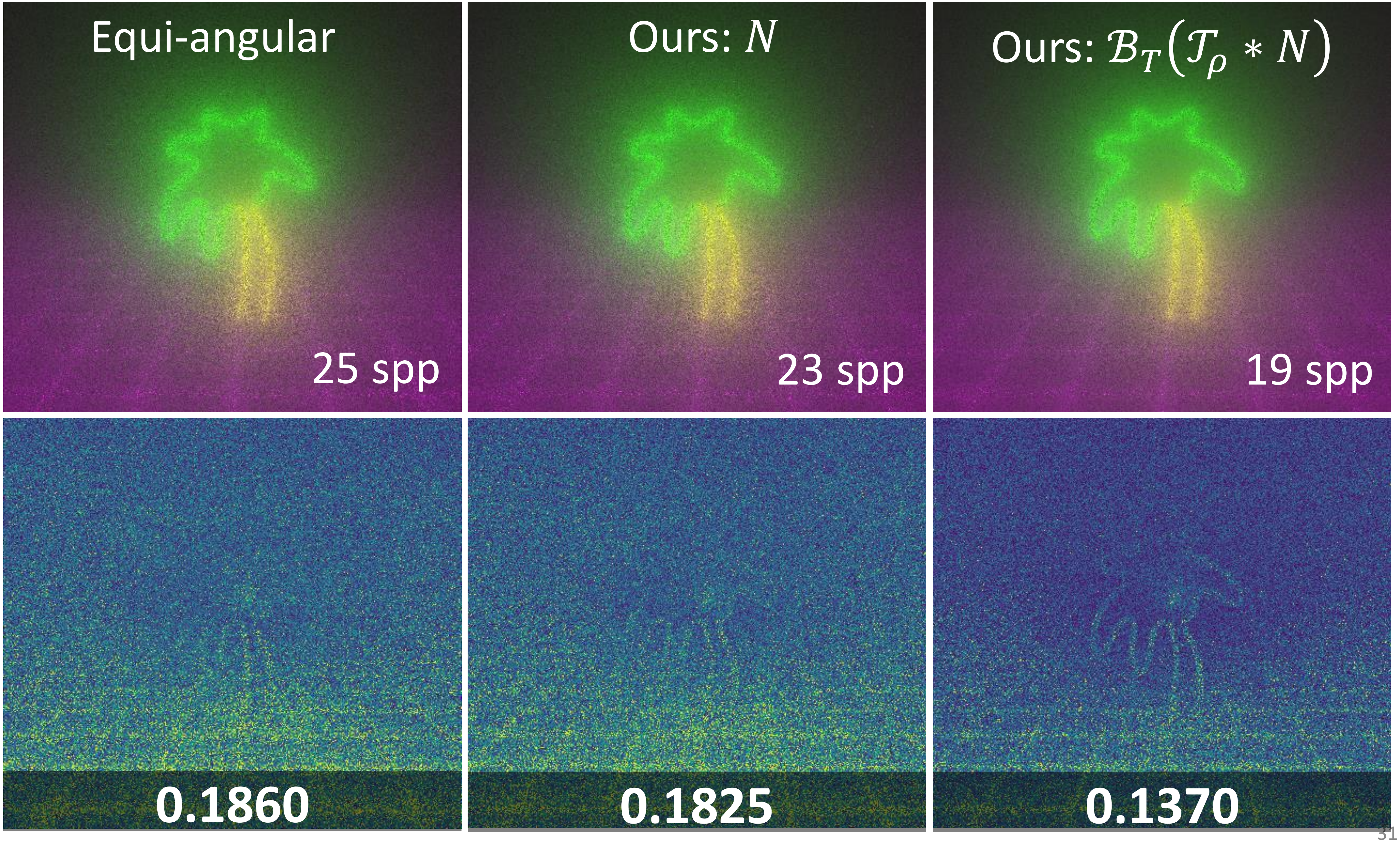
$$L = \frac{L_e}{h} \int_{\theta_{min}}^{\theta_{max}} \rho(\theta) T(\theta) N(\theta) d\theta$$

- Do Taylor Expansion of $\mathcal{T}_{T*P}$
- Uses [Hart et al. 2020]'s idea



$\mathcal{B}_\rho(\mathcal{T}_\mathrm{T} * N)$:	$\rho(\theta)$	$\mathcal{T}_\mathrm{T}(\theta) \times N(\theta)$
$\mathcal{B}_T(\mathcal{T}_\rho * N)$:	$T(\theta)$	$\mathcal{T}_\rho(\theta) \times N(\theta)$


Equi-angular
72 spp
0.262
Ours: $\mathcal{B}_T(\mathcal{T}_\rho)$
33 spp
0.038
Equi-angular
69 spp
0.143
Ours: $\mathcal{B}_T(\mathcal{T}_\rho * N)$
25 spp
0.045



Equi-angular
Ours: N
Ours: $\mathcal{B}_T(\mathcal{J}_\rho * N)$
25 spp
23 spp
19 spp
0.1860
0.1825
0.1370

Summary

- Summary:
  - New analytical method to sample cosine foreshortening and distance falloff
  - Approximate product sampling via Taylor expansion
  - Full approximate product with one warp composition

- Future work:
  - Specialize to planar lights, mesh lights, …
  - Extend to heterogeneous media
  - Handle refractive medium boundaries

Thank you for your attention :)